APPLICATION OF INFLUENCE DIAGRAM AND SEQUENTIAL HYPOTHESIS TESTING METHOD IN BRIDGE MAINTENANCE DECISION MAKING

Daifeng Hao
Department of Civil and Environmental Engineering
University of Missouri-Columbia

ABSTRACT

Bridge inspection is important in making maintenance or reconstruction decisions. Sometimes a single inspection fails to accurately reflect the actual condition, and additional inspections are needed. The paper addresses this issue from the perspective of minimizing the potential loss, that is, balancing the financial consequences of inspecting to obtain additional information and the financial consequences of making a decision with inadequate information.

An influence diagram is a tool used for decision support. The research described uses the framework of an influence diagram to build a bridge inspection and maintenance model. The sequential hypothesis testing method is a statistical approach used to estimate the expected cost of the whole decision process. In this paper, it is used to calculate the potential loss for the influence diagram model.

INTRODUCTION

There are more than 575,000 bridges over 20 feet in length in the United States recorded in the existing National Bridge Inventory (NBI) (1). Inspecting, maintaining, and repairing these bridges can be time-consuming and costly. More importantly, if an agency cannot detect an unsafe condition or does not make good decisions, the resulting price is perhaps not merely a waste of funds, but the loss of life.

The detection of construction deficiencies, deterioration, and subsequent damage depends on the inspection devices and methods used. The emergence of new technologies and the invention of better detection implements will narrow the difference between the detected condition and the actual condition. Yet, making good decisions on the time intervals of maintenance and rehabilitation (M&R) activities and the distribution of funds, which is much more complex, needs both expert experience and good data.

Errors during data collection are inevitable. Some errors are caused by inexperienced inspectors, while others are caused by inspection devices. Although researchers and engineers have developed some methods for dealing with the effects of uncertainty in data collection, one hundred percent accuracy of the data cannot be guaranteed. Inadequate data can cause cumulative effects in a series of decisions. However, too much data collection can result in the waste of money in bridge inspection and additional efforts spent on data processing. Therefore, determining the appropriate amount of data collection is an urgent task, and solving the problem can lead to improved decisions.

For example, assume an agency has used some data to determine whether a particular repair action should be applied to a bridge. The agency is then faced with the question of whether to collect a little more data to confirm the accuracy of the data already collected, or whether to apply the data in hand directly to decision making. In the first case, the agency should calculate the funds needed for additional data collection; in the latter case, the agency should estimate the cost of the risk for direct application of the existing data. Then, a comparison is made, and the lower cost alternative is the optimal choice. Such is a general illustration of the procedure for solving the problem

The research described uses influence diagrams to represent the bridge inspection and maintenance decision process. The sequential hypothesis testing method is applied, within the context of influence diagrams, to address the question of how much data to collect before making a maintenance or rehabilitation (M&R) decision. The paper describes bridge management, maintenance, and data collection practices, the combining of influence diagrams and the sequential hypothesis testing method, and the model developed.

DATA MANAGEMENT AND DATA NEEDS FOR DECISION MAKING

Bridge Management Systems

A bridge management system (BMS) is a tool that can help the agencies responsible for bridges make accurate decisions. BMS typically include inventory and condition data, and decision support algorithms. The National Bridge Inventory (NBI) contains information about the status of the nation’s bridges and magnitude of the funding needs. Yet, it cannot be classified as a BMS because it does not provide decision support. However, the NBI information can be used as the foundation for a BMS.

BMS at various levels of detail and completeness have been developed by a number of agencies. PONTIS and BRIDGIT are two commonly used BMS. PONTIS is designed to help agencies optimize network-level budgets and programs for maintaining and improving bridges (2). The approach PONTIS uses is “top-down”. Both budgets and standards are used to optimize policies, which are then used to select corresponding maintenance and rehabilitation (M&R) projects. BRIDGIT, which performs similar functions, was developed under the National Cooperative Highway Research Program (NCHRP) (3). BRIDGIT follows a “bottom-up” approach. In this approach, standards are used to plan projects, and these plans are used to estimate costs. The costs are compared with the budgets, and the result of the comparison is used to modify the original standards.

Bridge Maintenance Decision-making

In making bridge maintenance decisions, the bridge management agencies must decide how to allocate funds for M&R activities among a network of bridges and over time to maximize the funds' use. The common question concerning M&R decisions is: "which M&R activities should be performed on which bridges in the network?"

To answer the question, information on the current condition, which is provided by bridge inspection, is needed. In addition, the decision-maker needs forecast information on future condition, provided by a performance prediction model corresponding to specific bridges. The current condition information is used to support the decisions made on M&R activities. Therefore, the answer is not merely saying "yes" or "no" to the funds distribution, but also an explanation of "why" and a specification of "how".

In addition to deciding what M&R activities to take, the agency must also determine the frequency of M&R activities. While M&R frequencies can be defined at fixed intervals, sometimes M&R activities must be done in real time or at shorter intervals. For example, Cook and Lytton (4) addressed the issue of timing pavement M&R activities using a variable trigger point. Another group of researchers, Frangopol et al. (5), present the relationship between time and the cumulative maintenance cost, under different maintenance strategies.

The Federal Highway Administration uses the NBI to evaluate projects eligible for federal funds and to allocate these funds to the states. The current program is designed to target replacement activity as well as bridge rehabilitation activity, and the type of activity is based on the bridge rating obtained by inspection. The Highway Bridge Repair and Replacement Program (HBRRP) uses a discretionary bridge rating to allocate funds (6).

Data Collection

To obtain the information for the above appraisal, M&R decision–making, and prediction of bridge deterioration, data collection is essential. Before collecting data, the agency must determine what kind of data to collect. Therefore, the inspection should have a definite purpose. Depending on this purpose, data may or may not be recorded.

Different approaches have been used to collect different types of data. For example, visual inspection is used to collect surface data such as deck cracks; non-destructive testing or evaluation (NDT/NDE) is used to examine materials for structural integrity; and some destructive evaluation, such as burning, drilling, and grinding, is used to test chloride contamination level. For further discussion of these issues see (7).

Currently, BMS research is focused primarily on data collection methods and algorithm development to support the goal of optimal decisions for bridge maintenance and rehabilitation. For example, Turner and Richardson (8) describe bridge data needs and identify what data have the greatest impact on management decisions. Sanford and McNeil (9) address the broader issues in data collection, such as the role of geographic information systems (GIS) in linking BMS data with other data spatially, and the use of NDE in bridge inspection. Unfortunately, research on data collection and on decision support algorithms for management has been separate; that is, the algorithms assume the data collected are complete and error free. It is de facto not such a case. Therefore, further studies should concentrate on the integration of the two areas of research.

DATA REQUIREMENTS FOR BRIDGE MANAGEMENT

Managing data involves storing, filtering, and manipulating data throughout its life cycle. It is a series of processes to convert a large amount of raw data into formalized and useful information, so that the information can be used as the basis for appropriate decision-making. Components of data management for the decision support process include data collection, data preparation, data interpretation (analysis), and data application (Figure 1). Data requirements analysis determines whether there is a need to perform data collection. Generally speaking, most of the data in the BMS are renewed at 2-year intervals. But when some “special cases” occur, intermediate inspection is needed. Such “special cases” include a fracture found in the main structure of the bridge, or a scour critical bridge in a river with increasing water volume (10).

As stated above, manual and automated inspection are the two main methods for data collection. Because data collection can be costly, it is necessary for the decision-maker to determine what kind of data should be collected.

As data are obtained initially, they may be “noisy” or “dirty”. Therefore, data preparation is used to filter it. Filtering technologies include regression algorithms and neural networks. The former has the advantage of easy calculation, especially for linear modeling. It also has the disadvantage of imprecision. The major strengths of the latter are the ability to model complex and non-linear relationships, and the unrivaled ability at low-level pattern recognition. Its major weakness is the complexity in calculation (11).

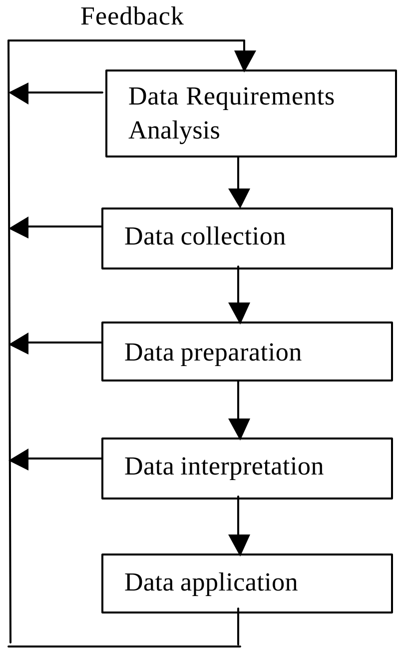


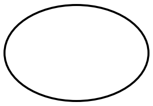
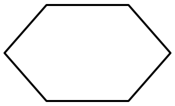
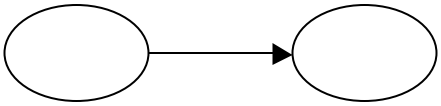
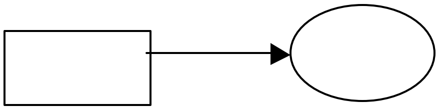
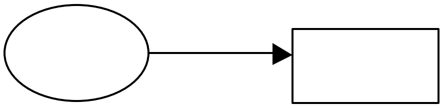
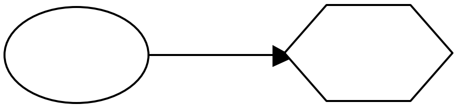
FIGURE 1. Data Management for Decision Support Process

Data integration utilizes methodology and technology to combine different types of data, or data in different periods, to derive more information. For instance, the determination of fatigue crack level could be integrated with Average Annual Daily Traffic (AADT), environmental factors, and material quality. Fatigue cracking is measured using NDE methods, which provide a quantitative measure of condition. However, current BMS are primarily designed to accept subjective condition ratings rather than measured data (9), and this leads to a flexibility or uncertainty in categorizing other types of data. Thus, as data integration provides tools to obtain information from BMS, it should focus on investigating opportunities for constructing future BMS to use more quantitative data as inputs (10).

An influence diagram, which represents a decision process at a particular point in time, can be used to determine the types of data that should be collected, based on the value the data provide. Nodes and arrows are the main components of an influence diagram (Table 1) (12, 13). In the diagram, three types of nodes are connected by arrows: chance nodes, decision nodes, and outcome nodes.

Influence diagrams clearly show the dependencies between data and the state of knowledge at a decision point, and they enable the evaluation of the outcome under different scenarios.

TABLE 1. Influence Diagram Components

Nodes and Arrow	Explanation
	Chance node indicates that an event could happen; its occurrence is beyond the control of the decision maker.
	Decision node indicates that a decision is made; it is under the control of the decision node.
	Outcome (also called objective in ANLYTICA) node indicates what the final result.
	Index node is used to help create table dimensions.
	The previous chance node affects the probability of the subsequent chance.
	The decision affects the probability of the subsequent chance.
	The decision is made knowing the probability of the chance occurrence.
	The occurrence of the outcome is contingent on the chance probability.
	The previous decision is made before the subsequent one.

SEQUENTIAL HYPOTHESIS TESTING METHOD

The sequential hypothesis testing method is a statistical tool used to derive the optimal scheme from several alternatives. Each alternative is represented by a hypothesis. Because a decision in one time period may affect the decision in the next time period, the choice of the optimal alternative for the whole time period is based on a Markov decision process. In each time period, the optimal value for different decisions is calculated and used as the input for the next time period. When the process terminates, the final optimal value is obtained, and the sequential decisions are determined. The core of Markov decision process is a Finite State Markov Chain which evolves over time. See (14) for additional details.

Research in this area has generated optimal strategies for bridge inspection and prediction of bridge deterioration rates. For example, Madanat (15) addresses the issues of making optimal decisions about pavement M&R under uncertainty by means of decision tools such as a decision tree. Mauch and Madanat (16) describe the use of survival modeling techniques to develop deterioration models.

As discussed, the inspection of a bridge deck is not error free. Therefore, all inspection and maintenance decisions should be coordinated so that the potential loss for the decisions is the minimum. Potential loss is the possible cost resulting from an incorrect decision. This can be accounted for by using a hypothesis testing model. The following formulation is adapted from (17).

In a sequential hypothesis testing model, after each measurement, the inspector is given the choice of either accepting one of the two hypotheses, or collecting an additional measurement prior to making a decision. That is, the inspector can

- accept the null hypothesis H_0 : the measurements are generated by f_0 ;
- accept the alternative hypothesis H_1 : the measurements are generated by f_1 ; or
- delay the decision until one more measurement is taken.

The distributions f_0 and f_1 describe a sequence of observations, which in this application represent the measurements of deterioration at different points on the bridge deck, in the substructure, or other part of the bridge.

The state of the information can be represented by the probability of H_0 being true, which is denoted by p_k . (It could also be based on the probability of it being false, because the two formulations are symmetric). Using Bayesian method:

$$p_{k+1} = \frac{p_k f_0(Z_{k+1})}{p_k f_0(Z_{k+1}) + (1 - p_k) f_1(Z_{k+1})}, \quad \text{k=0,1,…N-1,} \tag{1}$$

and

$$p_0 = \frac{pf_0(Z_0)}{pf_0(Z_0) + (1-p)f_1(Z_0)} \qquad (2)$$

where: p is the initial assessment of the probability of H_0 being true,

k represents the time period, and

Z_k represents the measurement of deterioration at different points on the bridge.

The term “loss” refers to the negative consequences of making an “incorrect” decision. Expected losses related to the selection among competing hypotheses are associated with two types of errors: type I error (reject the null hypothesis H_0 when it is true) will lead to loss L_0; type II error (accept the null hypothesis H_0 when it is false) will lead to loss L_1. Comparing these losses, and the cost of taking additional measurements to make a decision, the recursive expression for calculating the minimum expected cost ($\bar{J}_k(p_k)$) from the kth period until the end of the inspection process is:

$$\bar{J}_k(p_k) = \min[(1-p_k)L_0, p_k L_1, C + E\left\{\bar{J}_{k+1}\left[\frac{p_k f_0(Z_{k+1})}{p_k f_0(Z_{k+1}) + (1-p_k) f_1(Z_{k+1})}\right]\right\}] \qquad (3)$$

where: k=0,1,…N-2,

E is the expected value,

and

$$\bar{J}_{N-1}(p_{N-1}) = \min[(1-p_{N-1})L_0, p_{N-1}L_1] \qquad (4)$$

where: N is the time period, and

C is the inspection cost.

Therefore, the decision rule for the last period (N-1) is:

- If (1- p_{N-1}) $L_0 \leq p_{N-1}L_1 \Rightarrow p_{N-1} \geq \frac{L_0}{L_0 + L_1}$, then accept f_0.
- If $p_{N-1} \leq \frac{L_0}{L_0 + L_1}$, then accept f_1.

For the previous periods k=0…N-2, the optimal policy is also a threshold policy, given by:

- If $p_k \geq a_k$, then accept f_0.

- If $p_k \le \boldsymbol{b}_k$, then accept f_1.
- If $\boldsymbol{b}_k \le p_k \le \boldsymbol{a}_k$, then take one more observation.

where $\boldsymbol{a}_k$, $\boldsymbol{b}_k$ are determined from:

$$\boldsymbol{b}_k L_1 = C + E\{\bar{J}_{k+1}(p_{k+1})\} \tag{5}$$

$$(1-\boldsymbol{a}_k)L_0 = C + E\{\bar{J}_{k+1}(p_{k+1})\} \tag{6}$$

Furthermore, as $k \longrightarrow \infty$, $\boldsymbol{a}$ and $\boldsymbol{b}$ will converge to $\bar{\boldsymbol{a}}$ and $\bar{\boldsymbol{b}}$. Therefore, the optimal policy can be approximated by the stationary (time invariant) policy:

- If $p_k \ge \bar{\boldsymbol{a}}$, then accept f_0.
- If $p_k \le \bar{\boldsymbol{b}}$, then accept f_1.
- If $\bar{\boldsymbol{b}} \le p_k \le \bar{\boldsymbol{a}}$, then take one more observation.

To apply to the problem addressed by the paper, p corresponds to the overestimation probability; L_0 is the potential loss of skipping necessary M&R; L_1 is the potential loss of doing unnecessary M&R; C is the inspection cost; and f_0 and f_1 are functions that reflect the deterioration level under overestimation and underestimation, respectively. This is because as more inspections are made, the judgment should more accurately reflect the actual condition, so that the overestimation or underestimation probability is reduced.

RESEARCH METHODS AND TECHNIQUES

Influence Diagram for Bridge Inspection

An influence diagram has been constructed using ANALYTICA®, a tool that visually and quantitatively simulates decision making. The application software has the advantage of easily creating and displaying the structure of influence diagram, as well as doing mathematical calculation and analysis for each node. Figures 2, 3, 4, and 5 illustrate the decision model built in ANALYTICA, and Table 2 lists the node definitions.

FIGURE 2. Top level of Influence Diagram Module

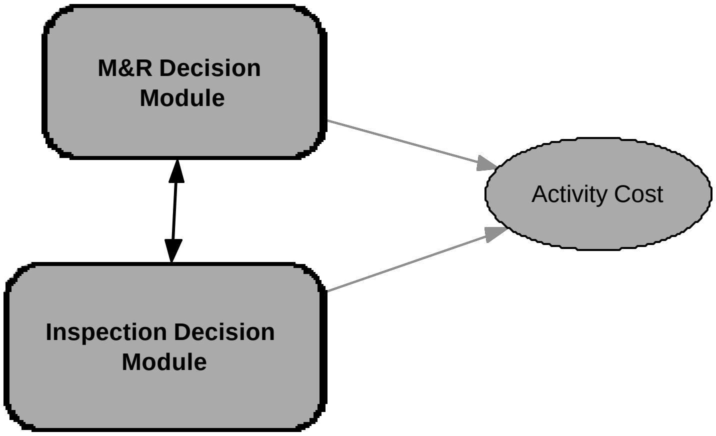


FIGURE 3. Middle level of Influence Diagram Module

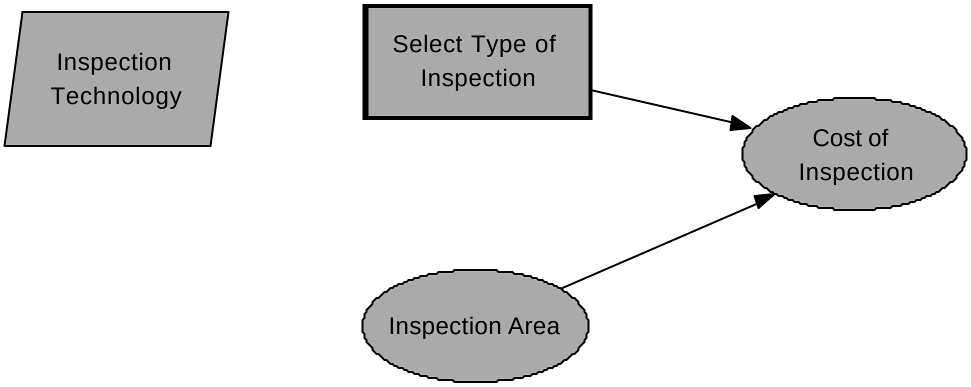


FIGURE 4. Lower level – Inspection Module

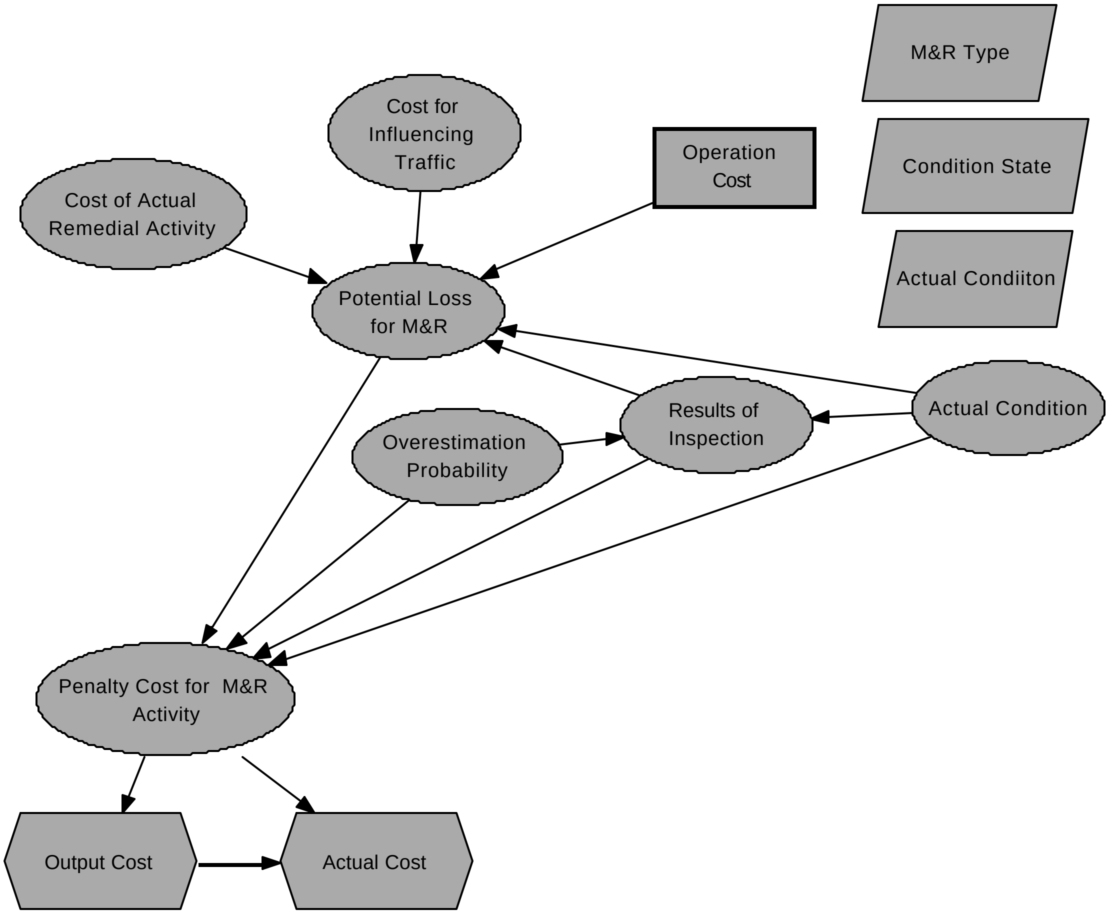


FIGURE 5. Lower level – Maintenance Module

TABLE 2. Influence Diagram Node Definitions

Node	Definition
Activity Cost (Objective)	The accumulating cost for each time period.
Actual Condition (Chance)	A condition state used by PONTIS.
Actual Condition (Index)	Combined with the condition states used by PONTIS, it gives the probability of how close the inspection results are to actual condition.
Actual Cost (Objective)	Cost value used in calculating activity cost.
Condition State (Index)	The bridge condition defined by PONTIS.
Cost for Influencing Traffic (Chance)	The cost because of traffic delays, obstructions, or detours. It can also be affected by the M&R action chosen or the inspection activity.
Cost of Actual Remedial Activity (Chance)	The cost for appropriate remedial action
Cost of Inspection (Chance)	The present inspection cost, which is determined by the type of inspection.
Expected Minimum Loss (Objective)	The calculated results of the optimal expected loss for this time period.
Inspection Area (Chance)	The area that the engineers inspect.
Inspection Technology (Index)	The inspection technology available in the module.
M&R Type (Index)	The types of maintenance that could be conducted.
Operation Cost (Chance)	The cost for different levels of maintenance.
Output Cost (Objective)	Combined with inspection cost, it is used to compare with previous M&R cost.
Overestimation Probability (Chance)	The probability of overestimating the actual bridge condition, i.e., the actual condition is worse than the inspection result.
Penalty Cost for M&R Activity (Chance)	The expected loss caused by doing maintenance and rehabilitation due to overestimating or underestimating the bridge condition.
Potential Loss for M&R (Chance)	The cost for M&R actions, no matter whether it is actually unnecessary because of overestimating or put off because of underestimating.
Results of Inspection (Chance)	The condition state provided by inspection engineers based on PONTIS rating standard.
Select Type of Inspection (Chance)	Determine what kind of inspection (visual inspection or NDE) should be conducted. Types of inspection include visual inspection and NDE evaluation.

The model consists of three levels of modules. The first level is the *decision module* (Figure 2). Its output is the *expected minimum loss*. The second level consists of two modules — *M&R decision module* and *inspection decision module* (Figure 3). Their output, *activity cost*, calculates the cumulative cost or potential loss for each step, i.e., the cost for inspection or the potential loss for making M&R decisions. The third level is the calculation of the *cost of inspection* and M&R potential loss (Figures 4 and 5). For the former, it is the multiplication of inspection area and the inspection cost per unit area, which is determined by the technology used. For the latter, it is more complex to determine the M&R potential loss. The core is the node *potential loss for M&R*. Before obtaining the expected penalty cost, the *actual condition* and *results of inspection* are compared. If the actual condition is poorer than the *results of inspection* indicate, that is, if the condition is overestimated, the penalty cost should be the potential loss for skipping necessary M&R multiplied by its *overestimation probability*. On the other hand, if the *actual condition* is better than the *results of inspection* indicate, that is, if the condition is underestimated, the penalty cost should be the *potential loss for doing unnecessary M&R* multiplied its underestimation probability, which is equal to one minus the *overestimation probability*. If the results of inspection match the actual condition, no penalty cost is generated. Two objective nodes are drawn from the penalty cost node. The *output cost* node is used for comparison, and the *actual cost* is used for computation.

Explanation of Models

Figure 6 shows the algorithm for inspection and maintenance decision making, and Table 3 illustrates the cost estimation process.

```
While not make M&R decisions
    If  time=1 then
    Do regular inspection;
    Activity cost[time]=cost of inspection;
    Else
        If cost of inspection[time-1]+output cost[time]>actual cost[time-1] then
            Do additional inspection;
            Activity cost[time]=activity cost[time-1]+cost of inspection[time];
        Else
            Reduce one inspection and do M&R; (*)
            Activity cost[time]=activity cost[time-1]-cost of inspection[time-1]
                                +actual cost[time-1];
            cost of inspection=0;
            actual cost=0;
        endif
    endif
    time=time+1;
endwhile
```

* one inspection is deducted because of the assumption that inspection is made before the M&R action is done.

FIGURE 6. Inspection and Maintenance Decision Algorithm

For the example in Table 3 (assuming that the inspection results are worse than the actual condition), when time equals one, the initial values for inspection, output cost, and actual cost are simulated to be set to, respectively, 10, 90, 90. Because the assumption is that inspection is always done before maintenance, the activity cost is set to be equal to the inspection cost 10. When time equals 2, because an inspection is done in the first time, the values for the three nodes should be 10, 64, 64. The output cost is obtained using the potential loss for M&R (the value is set to 450) times overestimation probability (initial value is set to 0.2), which is calculated recursively using formula (2). That is, as more inspection occurs, the potential loss should be deducted. Because the previous inspection cost plus the output cost is less than the previous actual cost (10+64<90), the activity cost is 10+10, which is 20. That means the decision to perform the initial inspection is correct. The second 10 is also the assumption that in the second time, an inspection will be performed before maintenance activity is done. Similarly, the third and fourth activity costs are 30 and 40, respectively. For the fifth time, because the previous inspection cost plus the output cost is no less than the previous actual cost (10+21>=31), the activity cost should be 40-10+31=60. This means that the assumption that inspection should be performed in the fourth time is incorrect; instead, maintenance should be done at the time. After that, all the inspection cost and actual cost is set to 0. After the process is over, the final activity cost is 61, and the activities performed are: inspection, inspection, inspection, and M&R. Figure 6 illustrates the simulation activity cost at each time step.

TABLE 3. Simulation of Influence Diagram Model

Time	Inspection cost	Output cost	Actual cost	Activity cost	Comments
1	10	90	90	10	Assume inspection is usually done in the first time
2	10	64	64	10+10=20	10+64<90
3	10	45	45	20+10=30	10+45<64
4	10	31	31	30+10=40	10+31<45
5	0	21	21	40-10+31=61	10+21>=31
6	0	14	0	61+0=61	0+14>0
7	0	10	0	61+0=61	0+10>0
8	0	6	0	61+0=61	0+6>0
9	0	4	0	61+0=61	0+4>0
10	0	3	0	61+0=61	0+3>0
Results	Therefore, the expected minimum loss is 61. Because the time latent reason, that is, we assume inspection each time, an inspection should be deducted. That means we need three inspections before we make final decisions.				

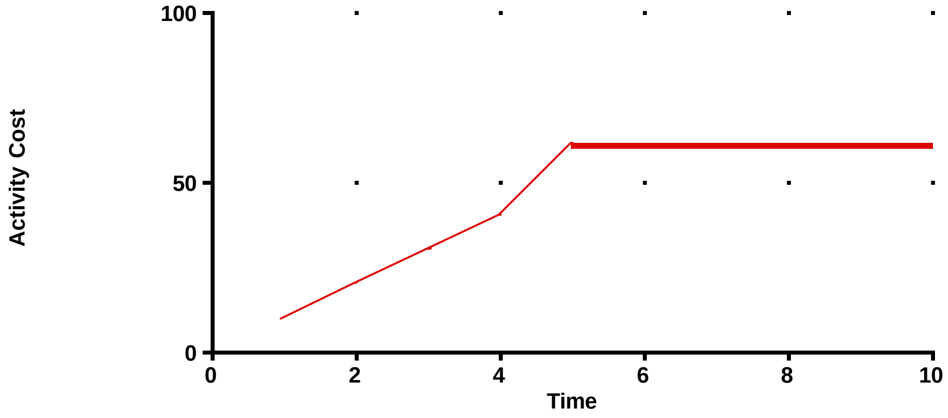


FIGURE 7. Model Simulation Results

CONCLUSION

The combination of influence diagrams and the sequential hypothesis testing method provides an approach for predicting the optimal number of inspections and estimating the minimum potential loss for the decisions. The model is appropriate for bridge maintenance decision making at the project level, i.e., the potential loss analysis for each bridge should be calculated individually. One problem to be noted in the model is that because of functional limitations of the software, the total number of inspections is always one more than the actual number of inspections.

Significant work remains. For instance, how can the definitions of the diagram nodes be modified so that they better reflect the actual decision process? In applying the approach to the network level of bridge inspection, what changes should be made to the influence diagram? How can a cost estimation model be converted into a bridge maintenance decision support system? And how could such a procedure be combined with existing BMS, such as PONTIS or BRIDGIT? Solving these problems will help to improve the bridge inspection and maintenance decision making process.

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