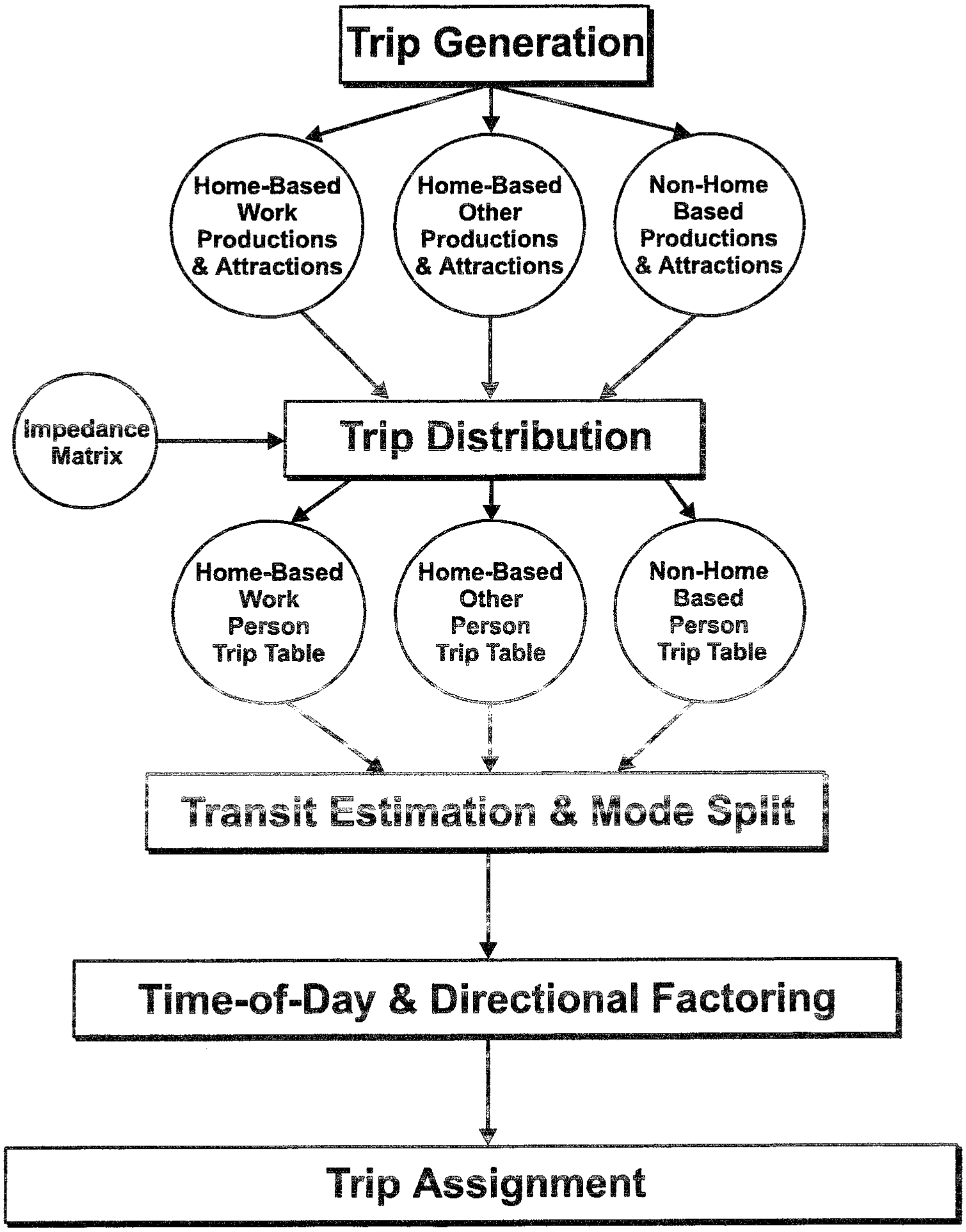
Session 6:
Trip Distribution



Session 6:
Trip Distribution

Objectives:

- Describe inputs and outputs to gravity model
- Explain concept of friction factors
- Explain where friction factors are obtained
- Apply gravity model to sample data set

Session 6:
Trip Distribution

Session Outline

- Terminology
- Key concepts
- Inputs and outputs
- Example problems
- Forecasting for future year assignments
- Error checking and validation

Session 6:
Trip Distribution

SESSION OUTLINE

This session outlines the process in TDF for determining where the trips will go. The session discusses key concepts such as the gravity model, calculating TAZ attractions, and friction factors. Due to its widespread use and limited time in the course, the only trip distribution technique to be discussed is the gravity model. Other models, such as the logit destination choice model, are used by some MPOs. The session also includes methodologies for forecasting for future year trip tables and example problems.

Notes:

Terminology

- Friction factor
- Gravity model
- K-factors
- Trip distribution

Terminology

Friction factor is a function of impedance of travel from production to attraction, measured in terms of travel time and cost. Friction factor (or the perception of distance) varies by trip type.

Gravity model is adapted from Newton's Law of Gravity. It requires estimates of the relative attractiveness of a TAZ as well as a measure of the impedances between TAZs.

K-factors are used to model individual zonal variation not otherwise accounted for in the gravity model.

Trip distribution uses trip ends from trip generation and network impedance to link trip ends to TAZs.

Notes:

Key Concepts

Trip distribution is a method to determine where the trips are going

- Distribute trips produced in one TAZ to all other TAZs
- Calibrate to reflect current travel patterns
- Apply to forecast future travel patterns

Inputs and Outputs

TRIP DISTRIBUTION

The trip distribution process uses data produced in the trip generation and path skimming steps of the TDF process. Travel times, or impedances, are used to measure the accessibility of a TAZ.

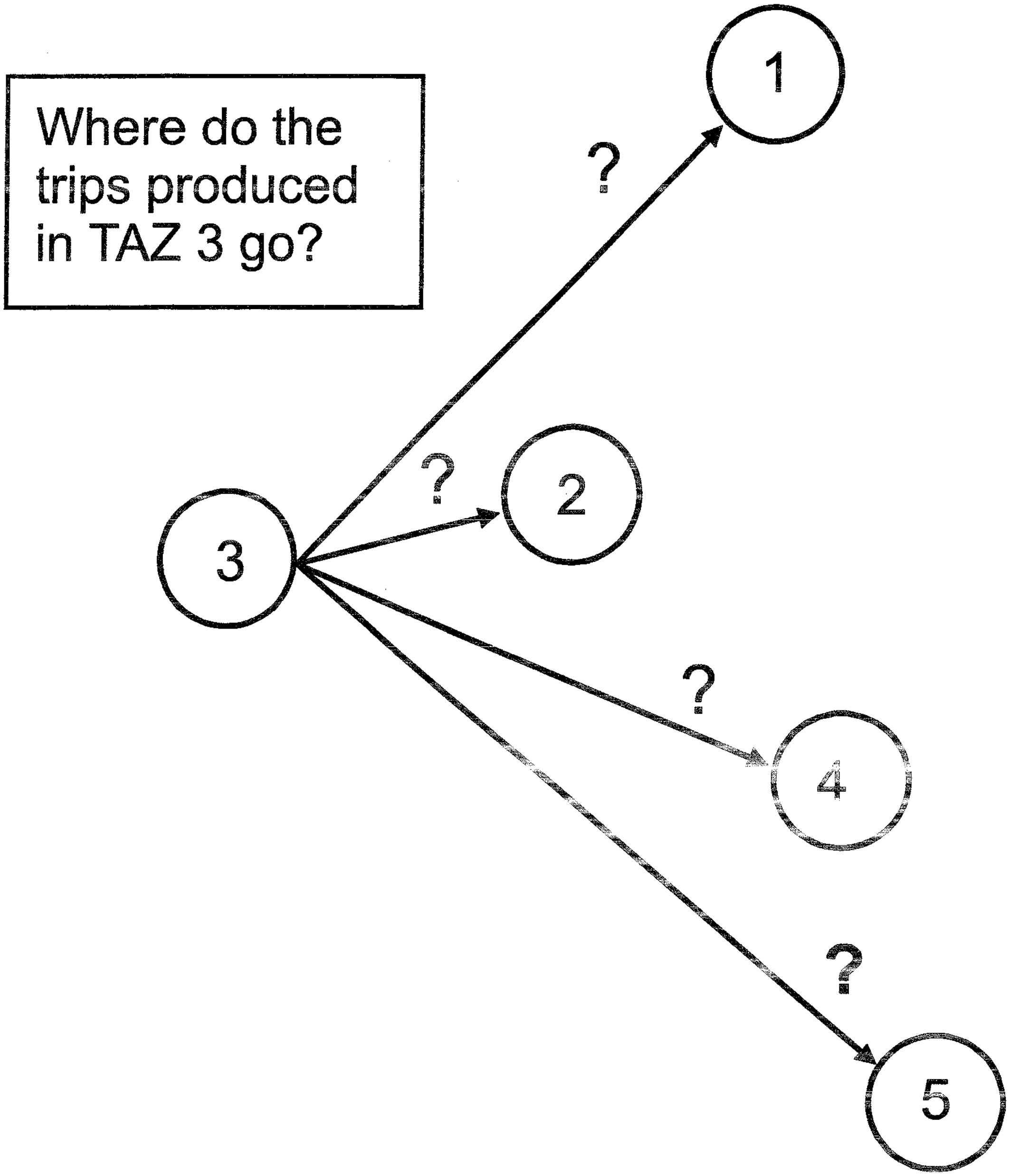
Travel times are in the form of a matrix; each cell represents the time it takes to travel from one TAZ to another TAZ. These numbers are derived from path skimming, which uses the transportation network to identify the shortest path to travel between each zonal interchange.

Productions and attractions are an output of trip generation. They are the number of trips produced from and attracted to each TAZ. The example in this session represents HBW productions and attractions, but HBO, NHB, and other trip purposes also will be calculated. It is important to note that the productions and attractions are "balanced," indicating that the total number of productions and attractions are equal for each trip purpose. For each trip produced in the study area, there is a related attraction. The balancing process was discussed earlier in Trip Generation (Session 5), and becomes important later in this session.

The output of the trip distribution model is a trip table. Each cell in the matrix represents the number of person trips between each zonal interchange.

Notes:

Gravity Model



Where do the trips produced in TAZ 3 go?

Gravity Model

The gravity model was adapted from Newton's Law of Gravitation, which states that the amount of gravitational force between masses is a function of the mass of the bodies and distance between them.

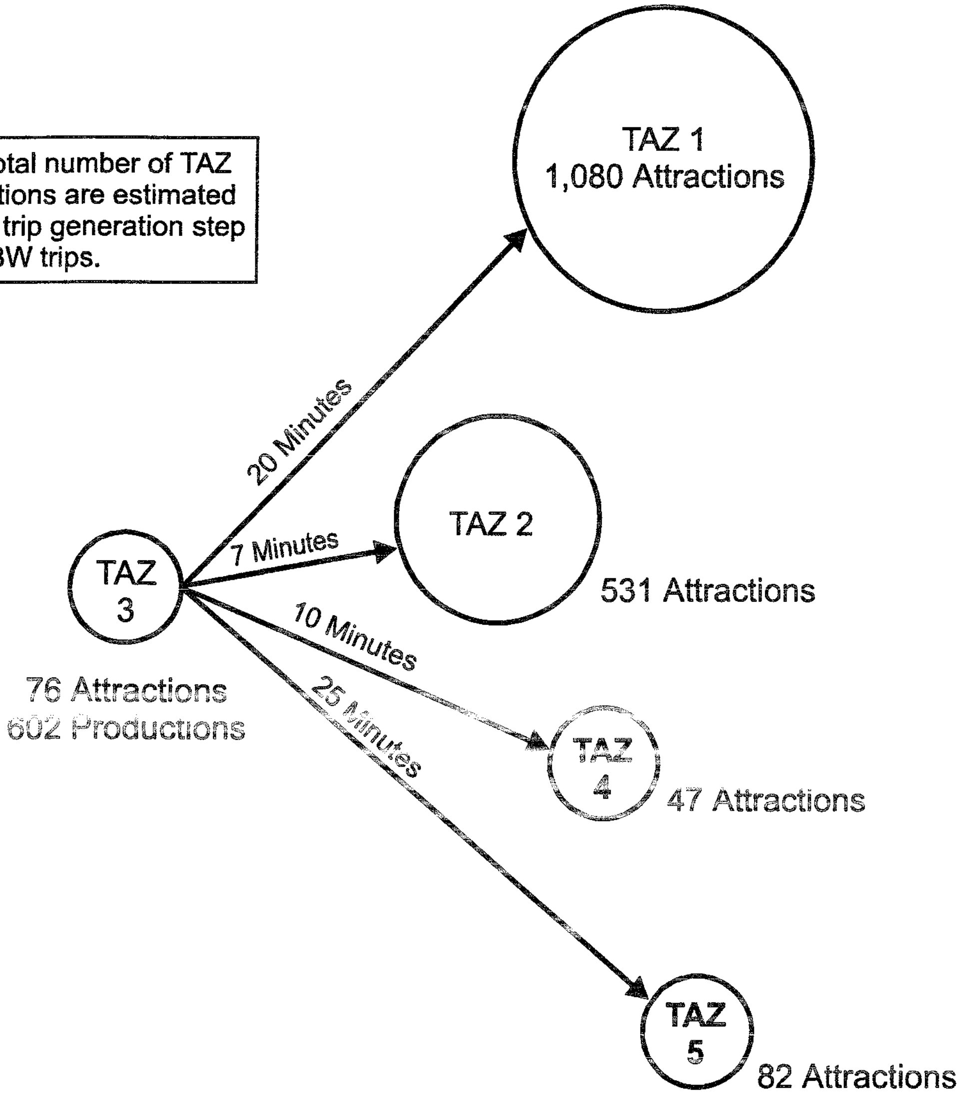
Again, the focus is to apply the gravity model to determine where trips go. The gravitational force in the gravity model will now become the amount of travel between TAZs. This application is possible if the calculations using the gravity model are based on the *relative attractiveness* of the zones. To apply the gravity model to travel, some modifications are needed. To convert the gravity model equation to represent the amount of travel or number of trips as opposed to gravitational force, two modifications must be made:

- accessibility is used instead of distance, and
- number of attractions is used instead of mass.

Notes:

Calculating TAZ Attractiveness

The total number of TAZ attractions are estimated in the trip generation step for HBW trips.



Calculating TAZ Attractiveness

Relative attractiveness is measuring how attractive a TAZ is when compared to all other TAZs.

The primary variables in calculating the relative attractiveness of a TAZ are:

- accessibility between TAZ, and
- number of attractions of the "TO" TAZ.

These two variables are represented in the figure for the distribution of productions from TAZ 3.

Accessibility

Accessibility is used in the equation because the more accessible a TAZ is, the more attractive it becomes. Accessibility is measured in the form of travel time or impedance (a combination of time and cost) that is derived from path skimming, discussed in the network development session. In the figure, the distance from TAZ 3 to the other TAZs represents travel time. While distance is relevant to travel choices, planners use the assumption that people will make a destination choice primarily based on accessibility or travel time as opposed to distance.

Number of Attractions

Even if a TAZ is near, if there is no reason to go there (no activities, services, jobs, or houses), it becomes less attractive. The number of attractions is used to represent activity of a TAZ. The figure shows that the TAZs are different sizes. This size difference represents the total number of attractions estimated for that particular TAZ in the trip generation step. The number of productions and attractions by zone for each trip purpose is an output of the trip generation step and is discussed in the trip generation session.

Notes:

Gravity Model

$$\text{Trips between TAZ 3 and "TO" TAZ} = \text{Trips produced in TAZ 3} \times \frac{\text{Attractiveness of "TO" TAZ}}{\text{Attractiveness of all TAZs}}$$

Gravity Model Equation

$$\text{Trips}_{ij} = \text{Productions}_i \times \frac{\text{Attractions}_j \times FF_{ij} \times K_{ij}}{\Sigma\ \text{Attractions}_j \times FF_{ij} \times K_{ij}}$$

Where:

Trips_{ij}	Trips between TAZs i and j
Productions_i	Productions from TAZ i
Attractions_j	Attractions to TAZ j
Friction Factor (FF_{ij})	Friction factor for TAZs i and j
Socioeconomic factor (K_{ij})	Socioeconomic factor for TAZs i and j

i=production TAZ
j=attraction TAZ

Gravity Model

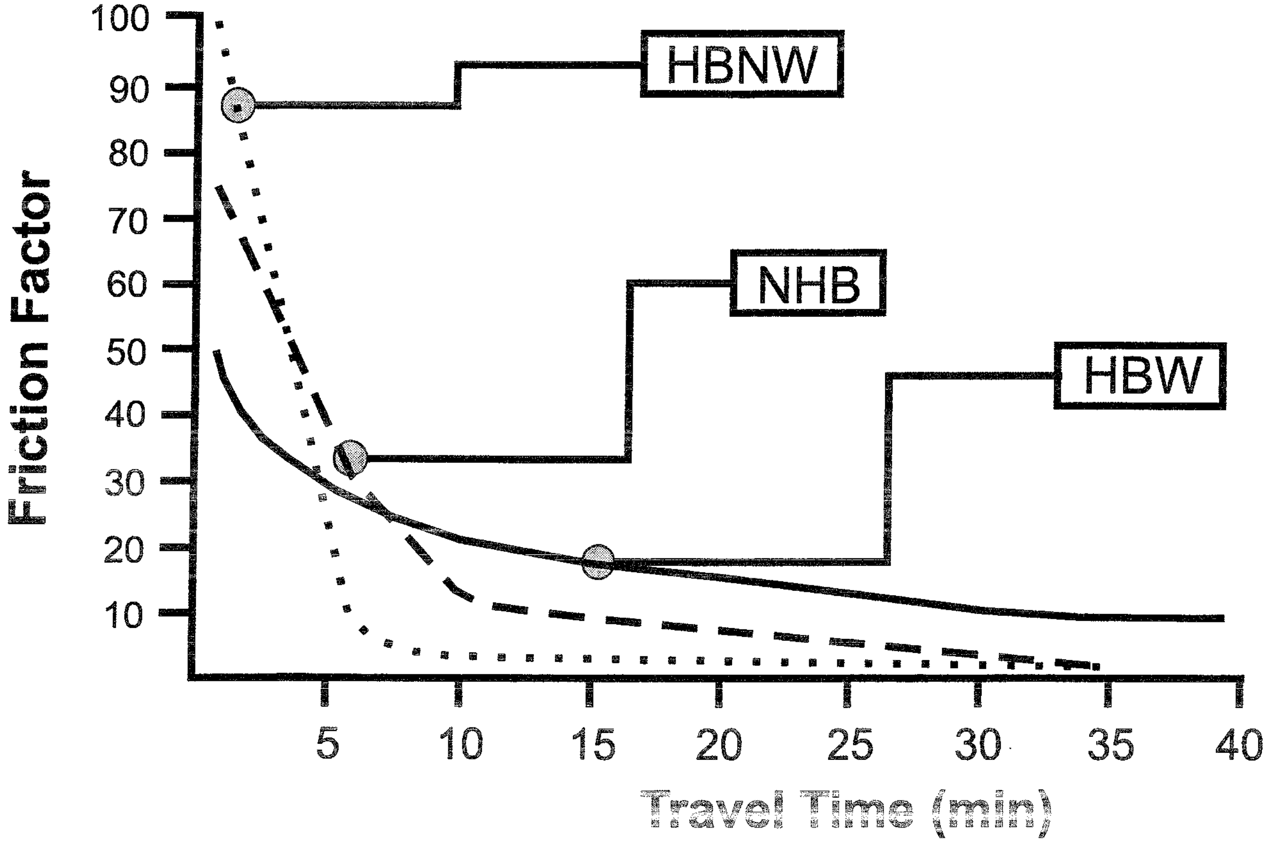
The gravity model determines the number of trips being exchanged between two TAZs. This calculation must be performed for all zonal interchanges in the region. For example, if a region has 500 TAZs, then 500 × 500, or 250,000, calculations are necessary. The variables in the equation are discussed below.

The right side of the gravity model represents the relative attractiveness of a TAZ. The greater the friction factor (FF_{ij}) or the number of attractions ($attractions_j$) compared to other TAZs, the greater the relative attractiveness of a TAZ.

Friction factors are used to represent travel time or impedance in the gravity model. These are discussed in detail next.

Notes:

Friction Factors



Friction Factors

Friction factors are used in the gravity model to express the effect of spatial separation or accessibility (travel time or impedance) on travel patterns.

Friction factors must be calibrated so that the frequency of travel, by length of trip, from trip distribution matches the frequency observed in the travel survey. Friction factors are calibrated using household travel surveys. The calibration process is illustrated in the session on calibration, validation, and reasonableness checks. Once calibrated, friction factors for a given travel time or impedance are assumed not to change for the forecast year.

The figure on the left shows that friction factors are higher as travel time decreases. A review of the gravity model helps explain this result. To calculate the relative attractiveness of a TAZ, the friction factor is multiplied by the number of attractions. The greater the friction factor or the number of attractions, the greater the relative attractiveness of a TAZ. This fact will become more clear after completing the trip distribution example.

Friction factor curves also differ by trip purpose because of differences in travel preferences. The average traveler will make decisions based on trip importance and the traveler's willingness to travel. Travelers are likely to spend more time making HBW trips than trips for other purposes. HBO trips usually are quite short; hence the higher friction factors on short trips. In other words, travelers are more likely to travel farther to get to jobs, but a HBO trip (grocery store, cleaners, or gas station) is more convenience driven. NHB trips (lunch, errands, and other trips taken during work hours) typically have friction factors between HBW and HBO.

Notes:

K-Factors

- K-factors account for socioeconomic linkages not accounted for by the gravity model
- Common application is for blue-collar workers living near white-collar jobs
- K-factors are i-j TAZ specific
- If i-j pair has too many trips, use K-factor less than 1.0
- If i-j pair has too few trips, use K-factor greater than 1.0
- Once calibrated, keep constant for forecast
- Use K-factors sparingly and with caution

K-Factors

Socioeconomic factors, or K-factors, consider defined social or economic linkages that affect travel behavior that are not otherwise considered in trip distribution. An example of these linkages would be a skills mismatch between skills required for jobs nearby and workers' skills in nearby neighborhoods. A specific example would be blue-collar workers living near white-collar jobs. The gravity model will distribute a large amount of blue-collar workers to the white-collar jobs since they are close, and there is a large amount of employment there. As a result, trips may be overestimated. If necessary, K-factors must be quantitatively related to socioeconomic characteristics of the particular zones to which they apply. These quantitative relationships are important since these factors are used in a forecast where the socioeconomic data will change.

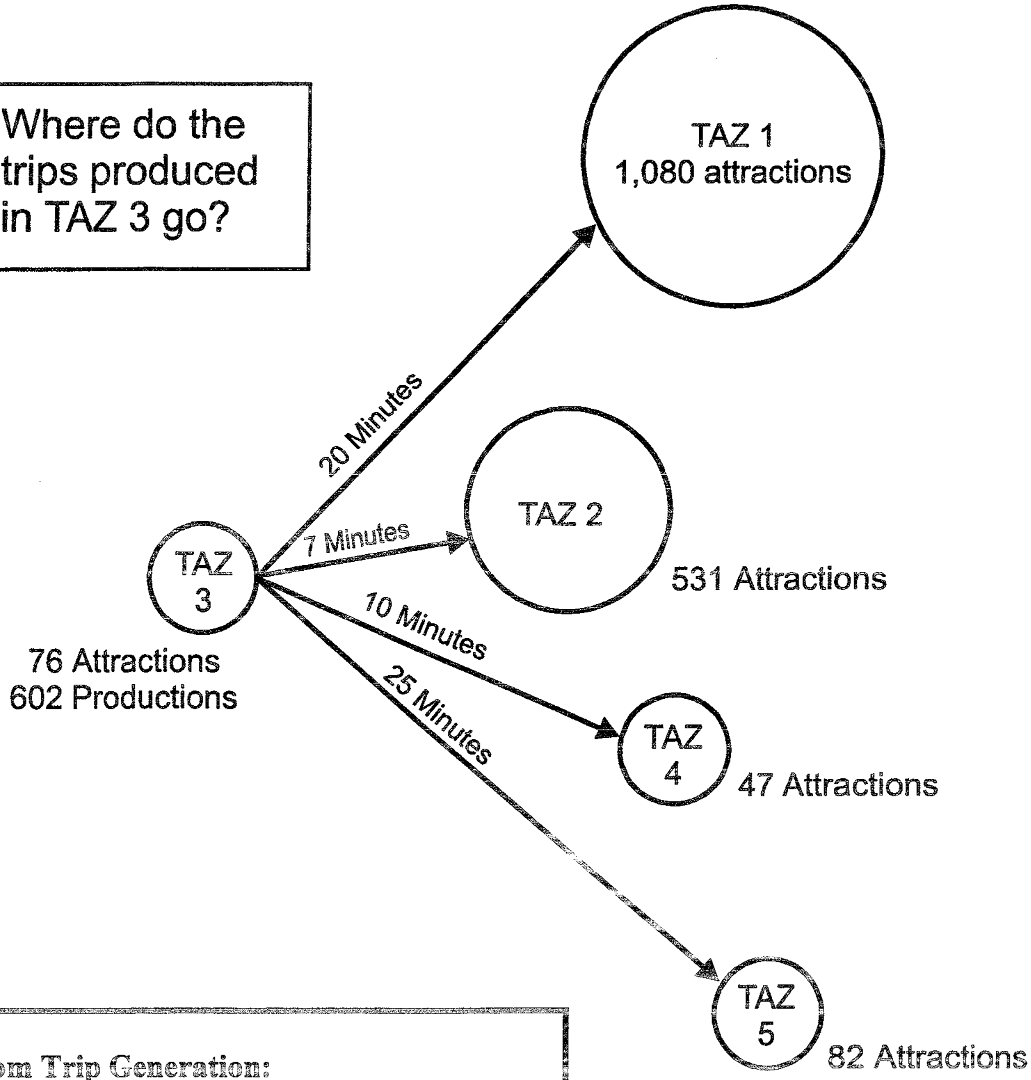
K-factors are applied to the gravity model as a fraction. If there is a deficiency of trips between TAZs and the condition cannot be corrected using calibrated friction factors, a K-factor greater than 1.0 would be applied, making the zone more attractive. If the zonal interchange has too many trips, a K-factor less than 1.0 would be applied. To prohibit trips, a zero is used.

The subjective nature of K-factors in application tends to reduce the credibility of the forecasts since the factors decrease the sensitivity of the model to variables that may change over time, such as changes in household incomes and skill levels. Some K-factors are justified, but in general, K-factors should be used sparingly and with caution. These factors will not be used in the class workshops or examples.

Notes:

Example Problem

Where do the trips produced in TAZ 3 go?



From Trip Generation:

HBW Productions and Attractions

TAZ	Productions	Attractions
1	234	**1080**
2	76	**531**
3	**602**	**76**
4	432	**47**
5	472	**82**

Example Problem

In this example, we will apply the gravity model by distributing productions from TAZ 3 to all the TAZs (Zones 1-5).

The figure depicts the number of attractions estimated for each zone by its varying size. The travel time from TAZ 3 to the other TAZs is also shown. These are the two primary variables in determining the relative attractiveness of a zone.

To apply the gravity model for HBW, the following data are required:

- HBW productions and attractions from trip generation;
- Travel times between TAZ 3 and all TAZs; and
- Friction factors.

The number of HBW productions and attractions for each TAZ are shown in the table. This table is the result of applying the work trip generation model and balancing the total number of HBW attractions to equal the total number of HBW productions.

These data are supplied here and on the following page.

Based on the diagram, which TAZ appears to be most attractive? Why?

Notes:

Example Problem

INPUT DATA

From Path Skimming

Travel Time Matrix

Attraction End

Production End

TAZ	1	2	3	4	5
1	4	12	8	15	21
2	6	3	9	23	14
3	20	7	4	10	25
4	12	18	8	4	17
5	24	19	23	15	8

HBW Friction Factors

Travel Time (min)	Friction Factor
3	87
4	45
7	29
10	18
15	10
20	6
25	4

Example Problem

INPUT DATA

The travel time matrix shows the acceptable travel time between all TAZ pairs. It also includes the intrazonal travel time for trips that are produced and attracted within the same TAZ. For example, the interzonal travel time from TAZ 3 to TAZ 5 is 25, and the intrazonal travel time is 4. Intrazonal travel time must be estimated since there is no network within TAZs. The most common method used is to take ½ the time to the nearest neighbor. For example, the estimated intrazonal travel time for TAZ 3 is ½ the time to the nearest TAZ (TAZ 2) = ½ × 7 = 3.5 min. The travel time matrix is typically the result of path skimming the highway network to find the minimum travel time and adding the terminal times for both ends of the trip.

For this example problem, the friction factors are provided. For example, the friction factor for an HBW trip of 25 minutes is 4, and the friction factor for an HBW trip of 4 minutes is 45. The HBW friction factors were calibrated for the base year so the resulting TLFD produced by the application of the gravity model matches the observed TLFD for the HBW trip purpose from the household travel survey.

Notes:

Example Problem

Convert Travel Times into Friction Factors

From Trip Distribution Calibrations:

Friction Factors

Travel Time (min)	Friction Factor
3	**87**
4	**45**
7	**29**
10	**18**
15	**10**
20	**6**
25	**4**

For TAZ 3:

Attraction TAZ	1	2	3	4	5
Travel Time	20	7	4	10	25
Friction Factor	**6**	**29**	**45**	**18**	**4**

Example Problem

CONVERT TRAVEL TIMES INTO FRICTION FACTORS

The top table shows a portion of a friction factor table. A complete friction factor table will show the friction factor for each minute of separation from one minute to the maximum permitted for a particular trip purpose (usually 45 to 90 minutes).

The bottom table shows one row of a travel time matrix—the row for trips produced in TAZ 3 and attracted to TAZs 1, 2, 3, 4, and 5. The associated friction factors are also shown (from the friction factor table above).

Notes:

Example Problem

Calculate the Attractiveness of Each Zone:

$\text{Attractiveness}_j = \#\ \text{Attractions}_j \times FF_{ij}$

i = production TAZ
j = attraction TAZ

Attraction TAZ→	1	2	3	4	5
# Attractions (A_j)	**1,080**	**531**	**76**	**47**	**82**
Friction Factor (FF_{ij})	**6**	**29**	**45**	**18**	**4**
Attractiveness (A_j * FF_{ij})	**6,480**	**15,399**	**3,420**	**846**	**328**

Which zone is the most attractive?

Example Problem

CALCULATE THE ATTRACTIVENESS OF EACH ZONE

The next step is to calculate the attractiveness of each TAZ. Attractiveness is the product of the number of attractions in the TAZ and a friction factor. In this example, the attractiveness of TAZ 1 is the product of the number of attractions in TAZ 1 (A_j = 1,080) and the friction factor for trips from TAZ 3 to TAZ 1 (FF_{ij} = 6). This product (1,080 × 6) is 6,480, which represents the attractiveness of TAZ 1 from TAZ 3.

Based on these calculations, which zone is most attractive? Which one will attract the most productions from TAZ 3?

Notes:

Example Problem

Calculate the Relative Attractiveness of Each TAZ

$$\text{Relative attractiveness} = \frac{(A_j \times FF_{ij})}{\Sigma\,(A_j \times FF_{ij})}$$

Attraction TAZ→	1	2	3	4	5	Sum of Attractiveness
Attractiveness $(A_j \times FF_{ij})$	6,480	15,399	3,420	846	328	26,473
Relative Attractiveness $\frac{(A_j \times FF_{ij})}{\sum (A_j \times FF_{ij})}$	**6,480/ 26,473 or 0.2448**	**0.5817**	**0.1292**	**0.0319**	**0.0124**	**1.0000**

Example Problem

CALCULATE THE RELATIVE ATTRACTIVENESS OF EACH TAZ

Relative attractiveness is a ratio of the attractiveness of one TAZ to the sum of the attractiveness for all TAZs.

To calculate the relative attractiveness of "TO" or j zone:

$$\text{Relative attractiveness} = \frac{(A_j \times FF_{ij})}{\Sigma\,(A_j \times FF_{ij})}$$

In this example, the attractiveness of TAZ 1 is 6,480, and the sum of the attractiveness for all TAZs is 26,473. The ratio of these two values is 0.2448 (6,480/26,473). The sum of the ratios will add to 1.000 (unless a rounding error occurs).

Notes:

Example Problem

Distribute Productions to TAZs

$$T_{ij} = P_i \times \frac{(A_i \times FF_{ij})}{\Sigma\,(A_j \times FF_{ij})}$$

TAZ	Productions from TAZ 3 $P_3 = 602$	Relative Attractiveness $\frac{(A_j \times FF_{ij})}{\sum (A_j \times FF_{ij})}$	Distributed Trips $P_i \times \frac{(A_j \times FF_{ij})}{\sum (A_j \times FF_{ij})}$
1	602	0.2448	**147**
2	602	0.5817	**350**
3	602	0.1292	**78**
4	602	0.0319	**19**
5	602	0.0124	**8**
Total	**1.000**		**602**

Example Problem

DISTRIBUTE PRODUCTIONS TO TAZ

Recall the gravity model:

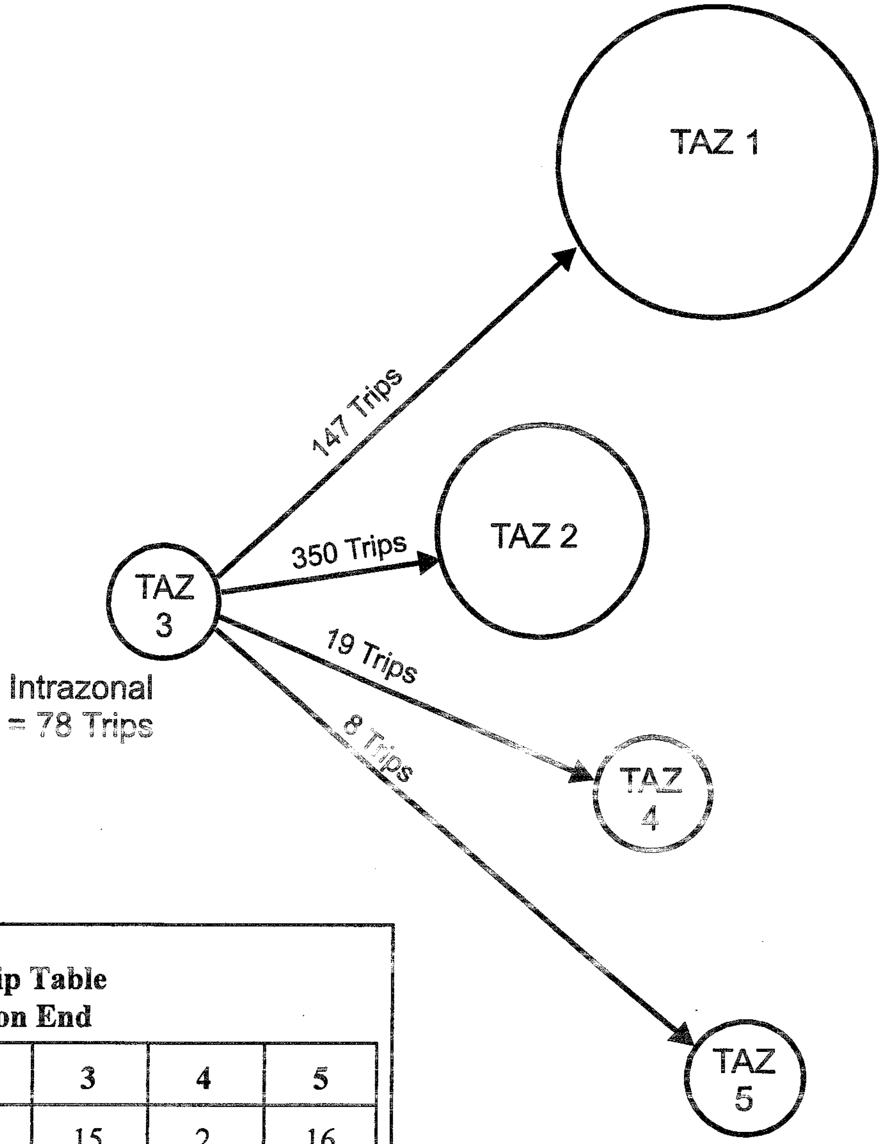
$$T_{ij} = P_i \times \frac{(A_i \times FF_{ij})}{\Sigma\,(A_j \times FF_{ij})}$$

To complete the distribution of TAZ 3 productions, apply the fraction of trips, or the relative attractiveness, to the number of TAZ 3 productions. This calculation will result in the number of trips distributed to each TAZ from TAZ 3.

Notes:

Example Problem

First Iteration Distribution



HBW Trip Table
Attraction End

Production End

TAZ	1	2	3	4	5
1	199	2	15	2	16
2	35	25	12	3	1
3	**147**	**350**	**78**	**19**	**8**
4	330	90	4	6	2
5	369	90	7	5	1

Example Problem

FIRST ITERATION DISTRIBUTION

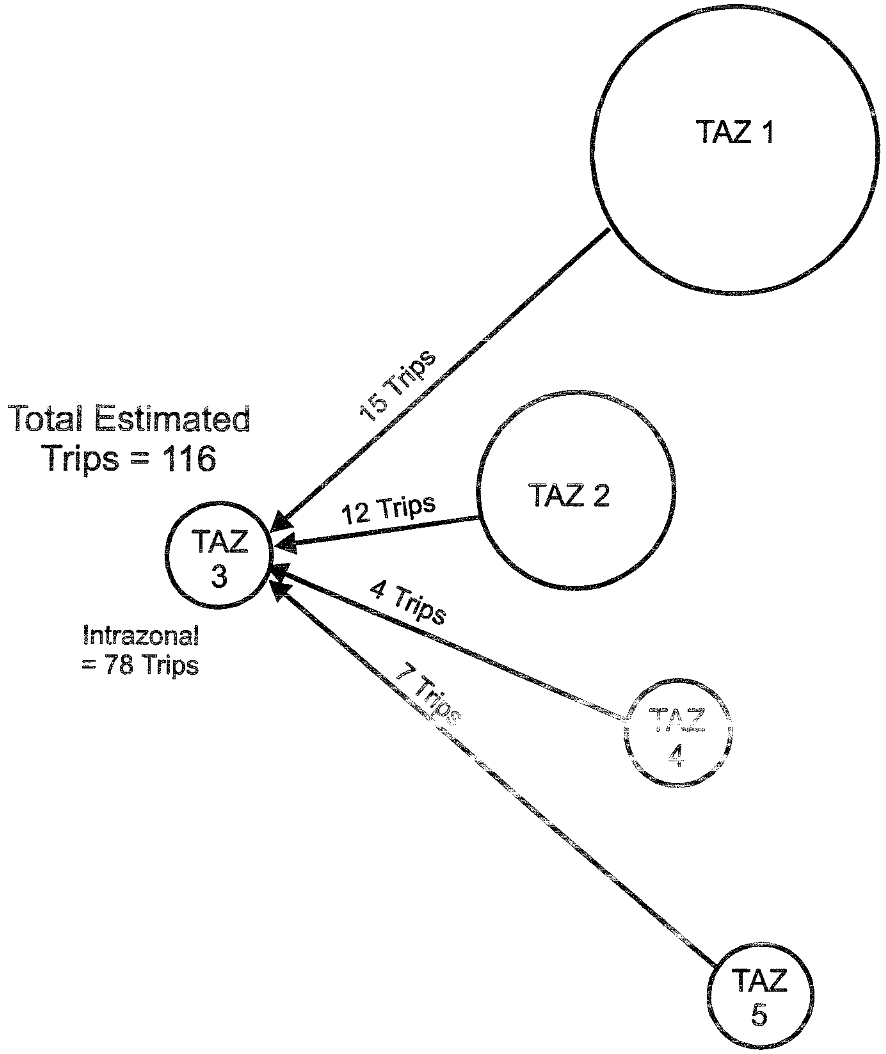
The results of the trip distribution example are shown on the figure. The trips distributed to each TAZ, including TAZ 3 (intrazonal trips), are shown.

The output of the trip distribution process is a trip table. At the bottom of the figure, a sample trip table is provided. The trips distributed in this example are highlighted in the table. The calculation performed in this example for TAZ 3 is repeated for TAZs 1, 2, 4, and 5.

Notes:

Example Problem

Comparing and Adjusting Zonal Attractions



Balanced attractions from trip generation = 76

The gravity model estimated more attractions to TAZ 3 than estimated by the trip generation model.

Example Problem

COMPARING AND ADJUSTING ZONAL ATTRACTIONS

The number of attractions used in the gravity model come from the trip generation step in the four-step process. The total production and attraction trip ends, by purpose, are balanced, meaning that for each HBW trip production end, there must be a HBW attraction end. To maintain this "balance," the number of attractions distributed to a TAZ must be checked against the number of attractions estimated in trip generation. A comparison is performed to ensure that these two numbers are in relative agreement.

To illustrate the balancing process, we compare the attractions for TAZ 3 from the trip generation step to the attractions for TAZ 3 from the trip distribution step. From the trip generation step, the attractions for TAZ 3 were estimated to be 76. From the trip distribution step, 116 trips were attracted to TAZ 3. The gravity model distributes trips based on the relative attractiveness of each TAZ. There is no control on the number of trips attracted to a TAZ. The attractiveness of TAZ 3 must be adjusted so that it will attract only 76 total trips.

On subsequent iterations, the number of attractions used in the gravity model (A_j) are adjusted by TAZ based on whether the gravity model over or under estimated trips attracted to that TAZ in the previous iteration. Iterations of the gravity model continue until the attractions from the gravity model and the attractions from trip generation match for each TAZ. Typically, four to five iterations are required.

Notes:

Forecasting for Future Year Assignments

- After successful base year calibration and validation
- Use forecast land use, socioeconomic data, system changes
- Forecasted production and attractions, and future year travel time skims
- Apply gravity model to forecast year
- Friction factors remain constant over time

Forecasting for Future Year Assignments

When the trip distribution model is calibrated and validated, the planner can apply it to a future year. The forecast requires forecasting land use, socioeconomic data, and anticipated transportation system changes in the network.

Information from the previous steps of the TDF process for the forecast year is used as input to the gravity model. This information includes productions and attractions from trip generation and travel times from path skimming. Friction factors are assumed not to change over time so that once calibrated as part of the base year model, they are not changed

Notes:

Calibration/Validation and Error Checking

Evaluate for Reasonableness:

- Observed and estimated trip lengths
- Observed and estimated TLFDs
- HBW trip distribution compared to Census JTW
- Forecast trip lengths by purpose
- Intrazonal trips

Calibration/Validation and Error Checking

Evaluation of the trip distribution step for reasonableness is difficult due to limited observed data. As a part of the base year model calibration, the modeled mean trip length and TLFDs by trip purpose are compared to the observed mean trip length and trip TLFDs by trip purpose. A judgement is made as to the reasonableness of these comparisons. If the modeled and observed mean trip lengths are within 5%, this would be considered reasonable. Also, comparisons are made between the travel model mean trip lengths by purpose and the reported mean trip lengths by purpose for other urban areas of similar geographical size. Typical values are reported in NCHRP Report 365, Travel Estimation Techniques for Urban Planning.

For the forecast years, the forecast mean trip lengths for the HBO and NHB trip purposes are expected to agree closely with the calibration year mean trip lengths. Work trip mean trip lengths tend to increase modestly over time if the geographical size of the urban area is increasing.

The HBW trip distribution from the gravity model can be compared with the 1990 or 2000 Census journey-to-work distribution. This is typically done by first aggregating the TAZs into larger districts or sectors and comparing the travel model and census trip interchange volumes or the percent of trip interchange volumes among the districts or sectors. A judgment is made as to the reasonableness of this comparison. A comparison is also made between the journey-to-work mean trip length and the travel model trip length for work trips. In 1990, the average travel time to work was 23.2 minutes.

An additional check is the ratio of the number of intrazonal trips and the total number of trips expressed as a percent. Typically, intrazonal trips comprise 3% to 5% of the total number of trips. This value depends on TAZ size.

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